Note Title

6.9 Design for High-Cycle Fatigue (HCF)

The objective of design in the HCF regime is to avoid fatigue failure altogether so that the part has an indefinite life. Our focus for the rest of this chapter is to apply what we have learned to design components in the HCF regime. Recall that we have divided variable loads into three categories: *Fully reversed*, *repeated*, and *fluctuating* as shown in Figure 6-6. In this figure:

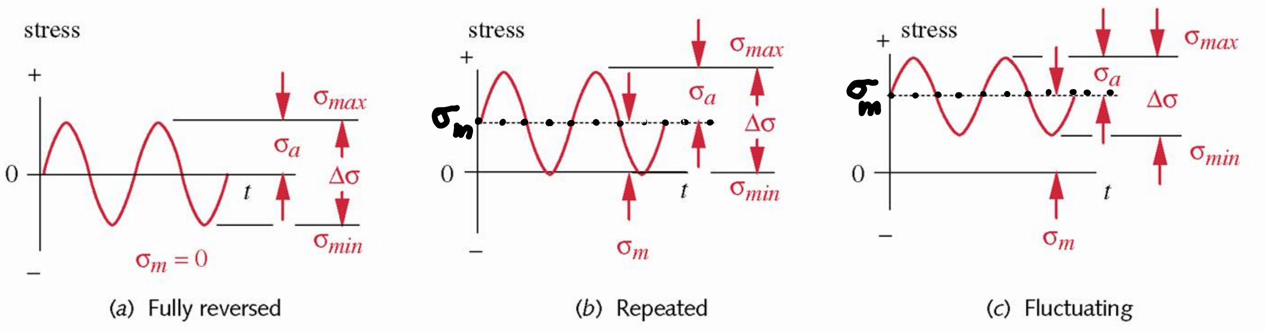


Figure 6-6
Alternating, Mean, and Range Values for Fully Reversed, Repeated, and Fluctuating Cyclic Stresses.

$$\sigma_m = \frac{1}{2}(\sigma_{max} + \sigma_{min}) \text{ and } \sigma_a = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$

are the mean and alternating stresses. In the fully-reversed case

$$\sigma_m = 0 \text{ but } \sigma_a \neq 0.$$

The design factor of safety for fatigue, N_f is given by

where

S_e or S_f = Corrected endurance limit or fatigue strength

σ' = The largest von Mises alternating stress at any location in the part, calculated to include all stress concentration effects.

When a tensile mean component of stress, σ_m is added to the alternating component, σ_a the material fails under a lower alternating stress than it does under fully reversed loading (Fig. 6-18, p. 324 of text). This is verified by observation as shown in Fig. 6-16.

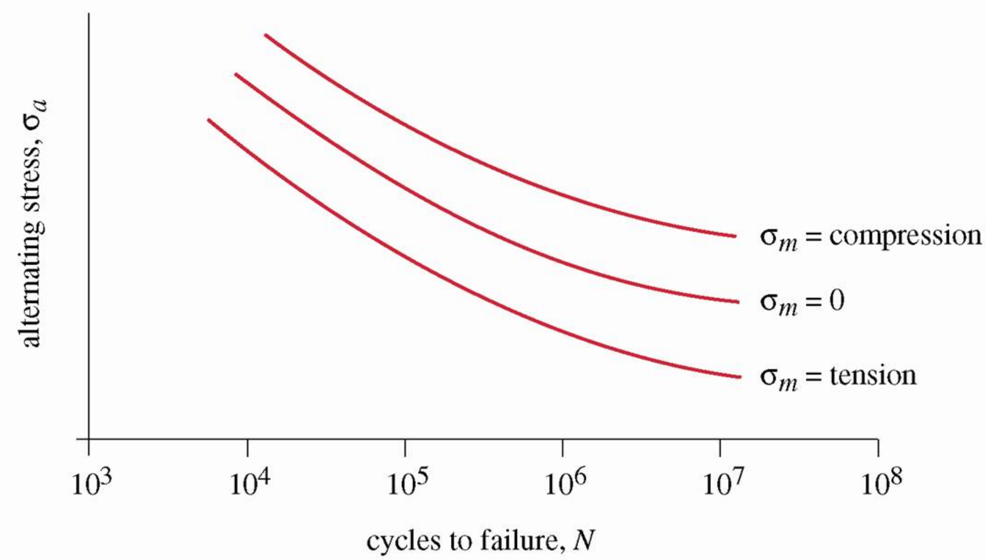


Figure 6-18

Effect of Mean Stress on Fatigue Life (From Fuchs and Stephens, *Metal Fatigue in Engineering*, New York, 1980, reprinted by permission of John Wiley & Sons, Inc.).

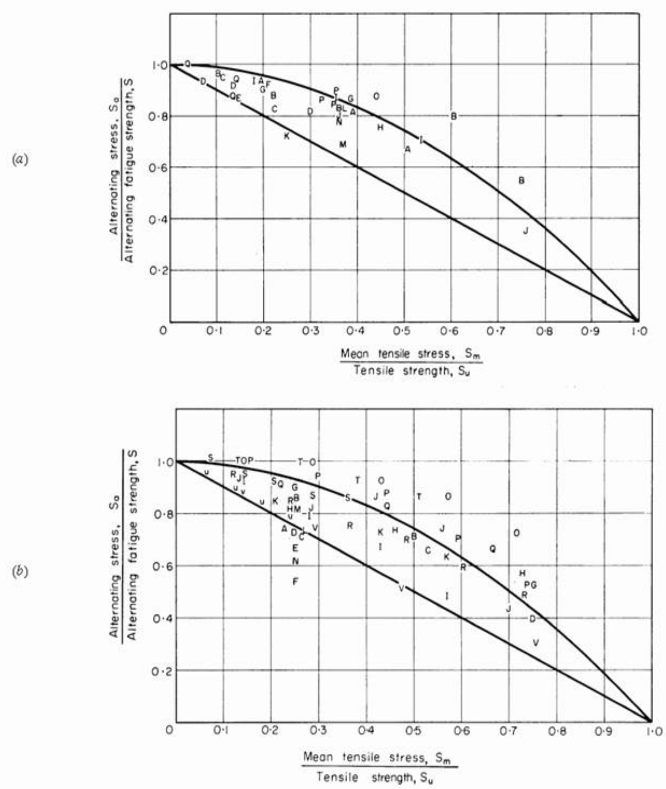


Figure 6-16

Effects of Mean Stress on Alternating Fatigue Strength at Long Life (*a*) Steels based on 10^7 to 10^8 Cycles (*b*) Aluminum Alloys Based on 5 x 10^8 Cycles. (From P. G. Forrest, *Fatigue of Metals,* Pergamon Press, London, 1962).

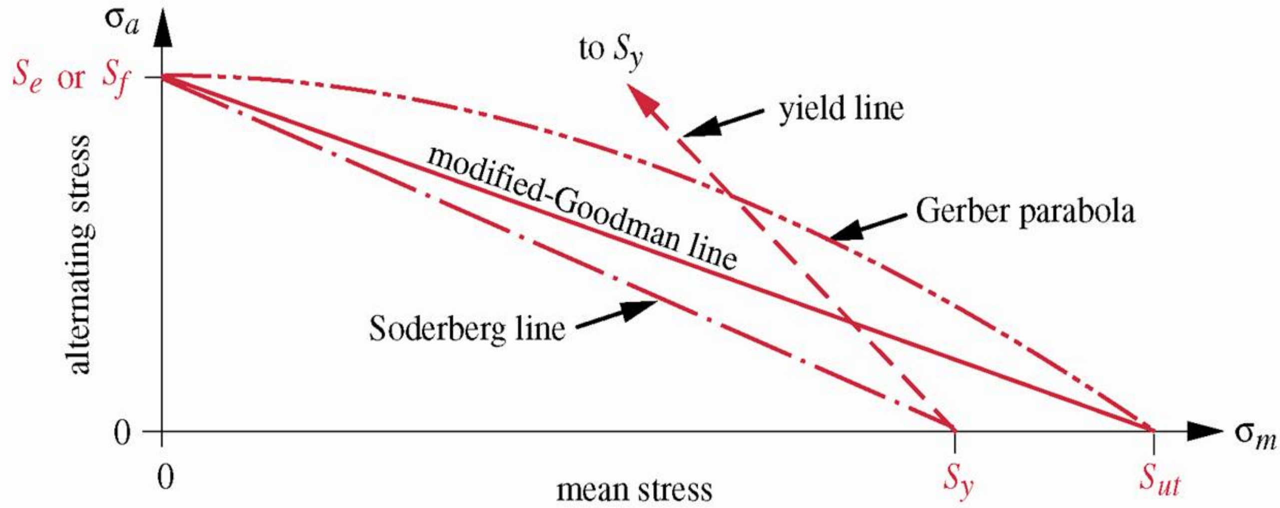


Figure 6-42
Various Failure Lines for Fluctuating Stresses.

Gerber parabola: $\sigma_a' = S_e\left(1 - \frac{\sigma_m'^2}{S_{ut}^2}\right)$ $(6.15a)_{mod}$

Modified-Goodman line: $\sigma_a' = S_e\left(1 - \frac{\sigma_m'}{S_{ut}}\right)$ $(6.15b)_{mod}$

Soderberg line: $\sigma_a' = S_e\left(1 - \frac{\sigma_m'}{S_y}\right)$ $(6.15c)_{mod}$

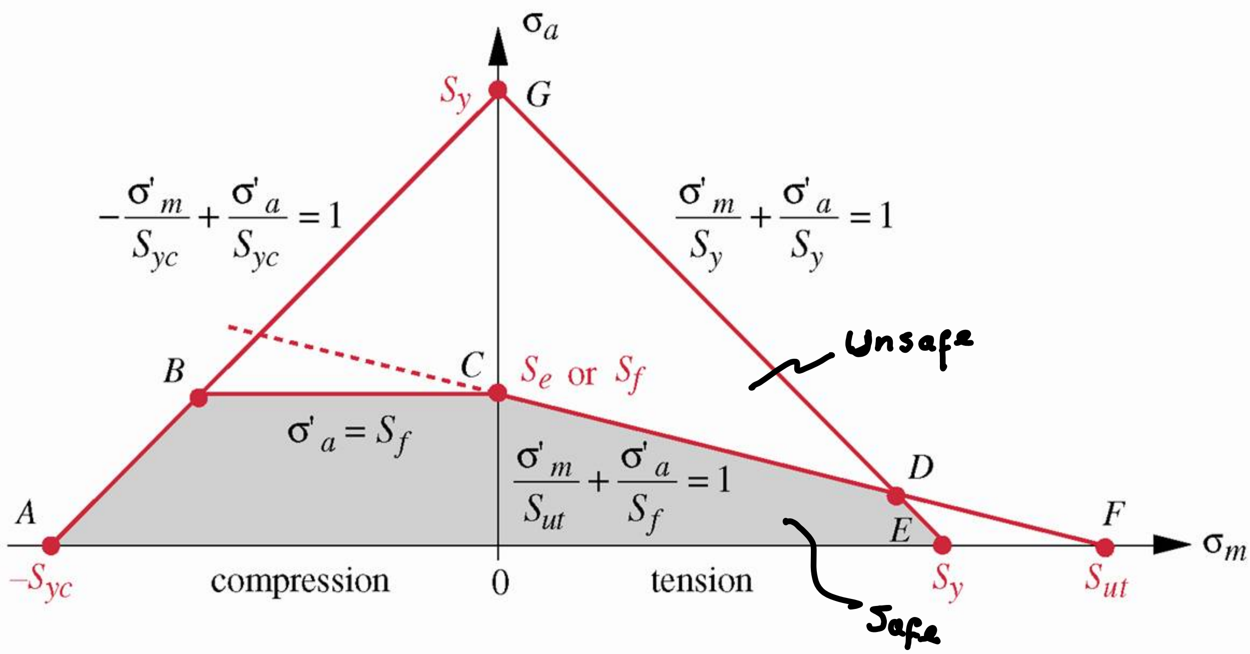


Figure 6-44
An "Augmented" Modified-Goodman Diagram.

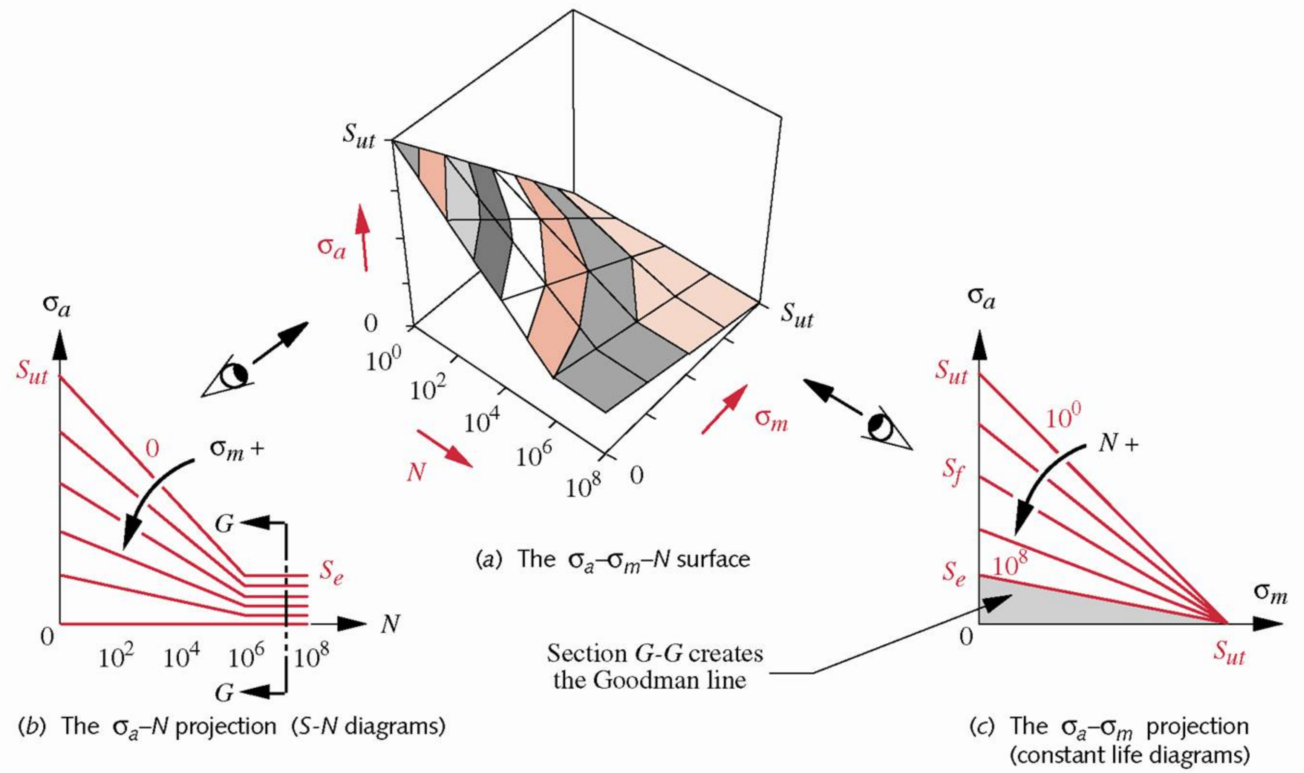


Figure 6-43
Effect of a Combination of Mean and Alternating Stresses.

Determination of the Factor of Safety

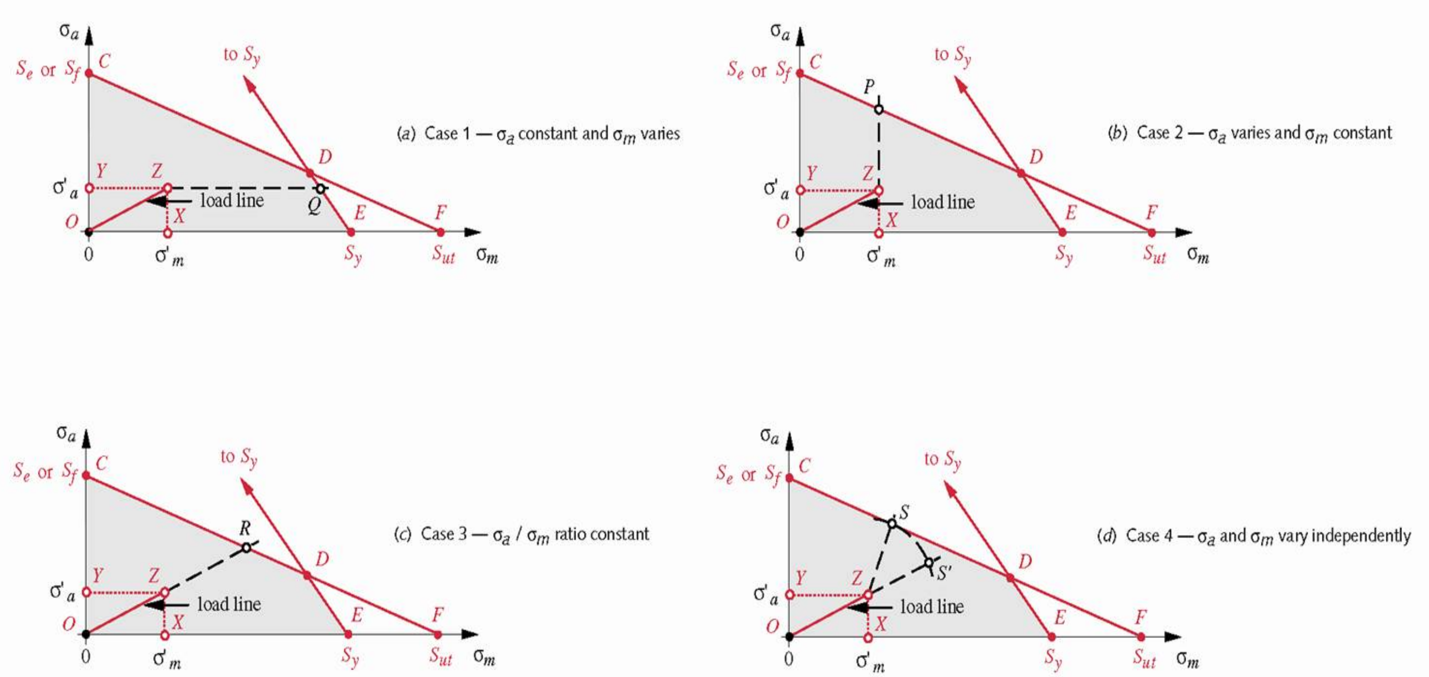


Figure 6-46
Safety Factors from the Modified-Goodman Diagram for Four Possible Load-Variation Scenarios.

Case 1:

$$N_f = \frac{\sigma'_m @ Q}{\sigma'_m @ Z}$$

where $$\sigma_m' @ Q = \begin{cases} \left(1 - \dfrac{\sigma_a'}{S_y}\right) S_y & \text{if } Q \text{ is on DE as shown} \\ \left(1 - \dfrac{\sigma_a'}{S_f}\right) S_{ut} & \text{if } Q \text{ is on CF} \end{cases}$$

and $$\sigma_m' @ Z = \sigma_m' \qquad \text{(this is the applied stress)}$$

$$\therefore \quad N_f = \begin{cases} \dfrac{S_y}{\sigma_m'}\left(1 - \dfrac{\sigma_a'}{S_y}\right) & \text{if } Q \text{ is on DE as shown} \\ \dfrac{S_{ut}}{\sigma_m'}\left(1 - \dfrac{\sigma_a'}{S_f}\right) & \text{if } Q \text{ is on CF} \end{cases} \qquad (6.18a)$$

Case 2:

$$N_f = \frac{\sigma_a' @ P}{\sigma_a' @ Z}$$

where $$\sigma_a' @ P = \begin{cases} \left(1 - \dfrac{\sigma_m'}{S_{ut}}\right) S_f & \text{if } P \text{ is on CD as shown} \\ \left(1 - \dfrac{\sigma_m'}{S_y}\right) S_y & \text{if } P \text{ is on DE} \end{cases}$$

and $$\sigma_a' @ Z = \sigma_a' \qquad \text{(this is the applied stress)}$$

$$\therefore \quad N_f = \begin{cases} \dfrac{S_f}{\sigma_a'}\left(1 - \dfrac{\sigma_m'}{S_{ut}}\right) & \text{if } P \text{ is on CD} \\ \dfrac{S_y}{\sigma_a'}\left(1 - \dfrac{\sigma_m'}{S_y}\right) & \text{if } P \text{ is on DE} \end{cases} \qquad (6.18b)$$

Case 3:

$$N_f = \frac{\sigma_m' @ R}{\sigma_m' @ Z}$$

where $$\sigma_m' @ R = \begin{cases} \dfrac{S_f}{\dfrac{\sigma_a'}{\sigma_m'} + \dfrac{S_f}{S_{ut}}} & \text{if } R \text{ is on CD} \\ \dfrac{S_y}{\dfrac{\sigma_a'}{\sigma_m'} + 1} & \text{if } R \text{ is on DE} \end{cases} \qquad (6.18d)$$

and

$$\sigma'_m @ Z = \sigma'_m \qquad \text{(this is the applied stress)}$$

or

$$N_f = \begin{cases} \dfrac{S_f\, S_{ut}}{\sigma'_a\, S_{ut} + \sigma'_m\, S_f} & \text{if } R \text{ is on } CD \\[2ex] \dfrac{S_y}{\sigma'_a + \sigma'_m} & \text{if } R \text{ is on } DE \end{cases} \qquad (6.18e)$$

Case 4:

$$N_f = \frac{\overline{OS}'}{\overline{OZ}} = \frac{\overline{OZ} + \overline{ZS}}{\overline{OZ}} \qquad (6.18g)$$

where $\overline{OZ} = \sqrt{(\sigma'_a)^2 + (\sigma'_m)^2}$

$$\overline{ZS} = \sqrt{(\sigma'_m - \sigma'_m @ S)^2 + (\sigma'_a - \sigma'_a @ S)^2} \qquad (6.18f)$$

and where

$$\sigma'_m @ S = \frac{S_{ut}\left(S_f^2 - S_f\sigma'_a + S_{ut}\sigma'_m\right)}{S_f^2 + S_{ut}^2}$$

$$\sigma'_a @ S = -\frac{S_f}{S_{ut}}\left(\sigma'_m @ S\right) + S_f$$

There is also the possibility that point S lies on line DE instead of CD in which case Eq. 6.16d should be used in place of 6.16c to find N_f.

Note that σ'_a and σ'_m in Eq. (6.18) are the von Mises alternating and mean applied stresses respectively:

$$\sigma'_a = \sqrt{\tfrac{1}{2}\left[(\sigma_{xa} - \sigma_{ya})^2 + (\sigma_{ya} - \sigma_{za})^2 + (\sigma_{za} - \sigma_{xa})^2 + 6(\tau_{xya}^2 + \tau_{yza}^2 + \tau_{zxa}^2)\right]}$$

$$\sigma'_m = \sqrt{\tfrac{1}{2}\left[(\sigma_{xm} - \sigma_{ym})^2 + (\sigma_{ym} - \sigma_{zm})^2 + (\sigma_{zm} - \sigma_{xm})^2 + 6(\tau_{xym}^2 + \tau_{yzm}^2 + \tau_{zxm}^2)\right]} \qquad (6.22a)$$

or, for a biaxial stress state:

$$\sigma'_a = \sqrt{\sigma_{xa}^2 + \sigma_{ya}^2 - \sigma_{xa}\sigma_{ya} + 3\tau_{xya}^2}$$

$$\sigma'_m = \sqrt{\sigma_{xm}^2 + \sigma_{ym}^2 - \sigma_{xm}\sigma_{ym} + 3\tau_{xym}^2} \qquad (6.22b)$$

Applying the Stress-Concentration Effects

The alternating component of stress, σ_a is treated as before. The alternating component of stress, σ_a is treated as before. That is, K_t is found first. Then q is determined, and finally K_f is derived from Eq (6.11b): $K_f = 1 + q(K_t-1)$. The local value of σ_a is then obtained from $\sigma_a = K_f\,(\sigma_a)_{nom}$.

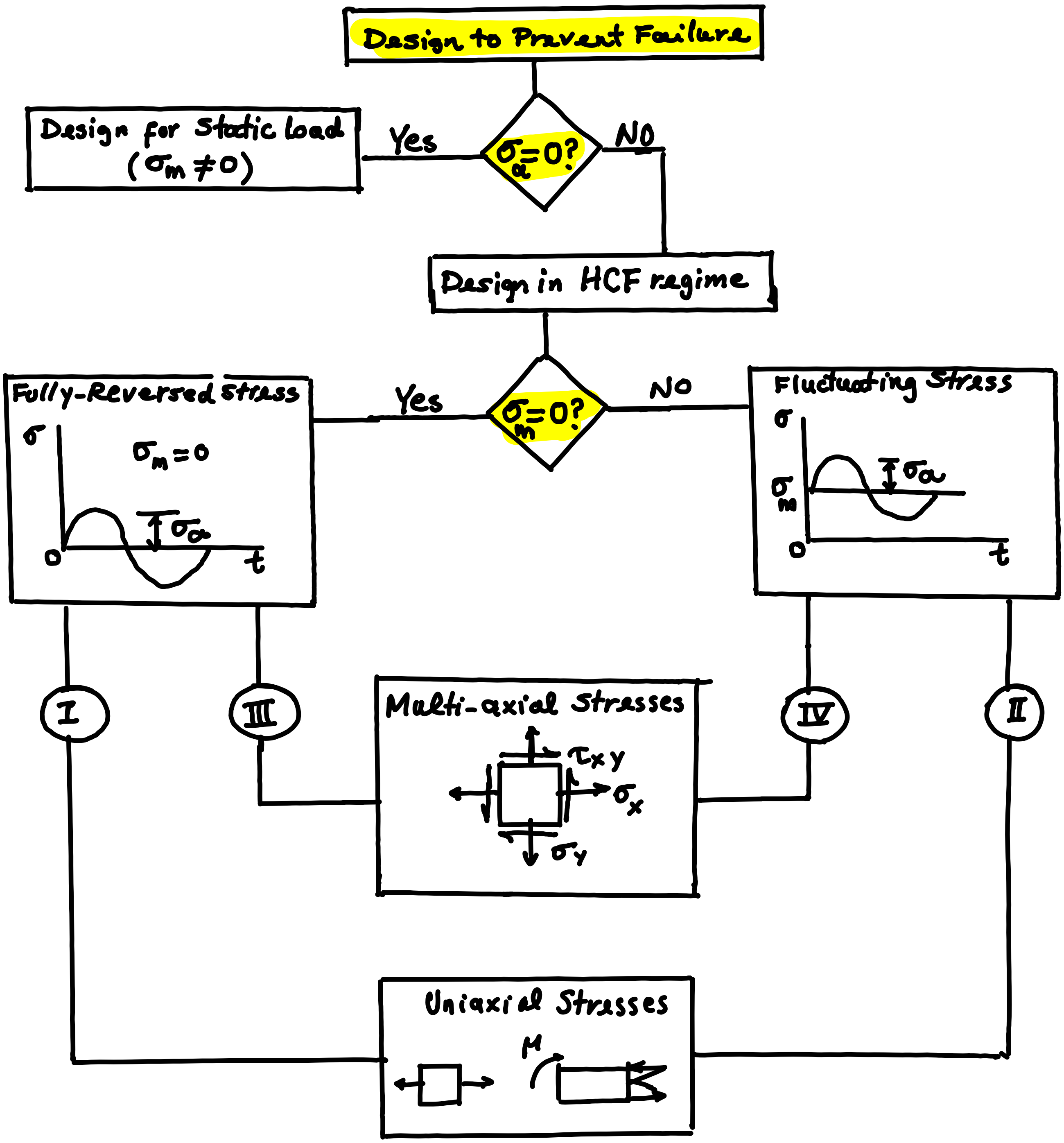
The mean component of stress is treated differently depending on the ductility or brittleness of the material.

In summary (see Eqs. 6.17, page 364):

$$\text{if} \quad K_f\,|(\sigma_{max})_{nom}| < S_y \qquad \text{then} \qquad K_{fm} = K_f$$

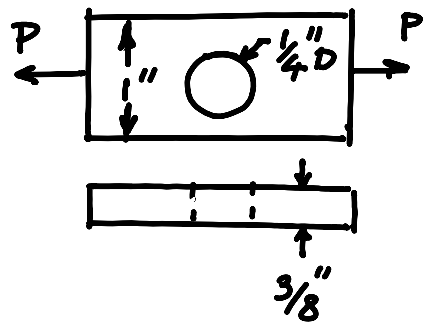
$$\text{if} \quad K_f\,|(\sigma_{max})_{nom}| > S_y \qquad \text{then} \qquad K_{fm} = \frac{S_y - K_f\,(\sigma_a)_{nom}}{|(\sigma_m)_{nom}|}$$

$$\text{if} \quad K_f\,|(\sigma_{max})_{nom} - (\sigma_{min})_{nom}| > 2S_y \qquad \text{then} \qquad K_{fm} = 0$$



As shown in the above chart there are four types of problems that can arise. However, by using the von Mises effective stress in the Modified Goodman Line (GML) the relations obtained could be applied to all four types of problems.

Example: The cold-rolled. AISI 1020 steel bar shown is subjected to a tension load between 800 and 3000 lb. Estimate the factor of safety guarding against failure by yielding and failure by fatigue.

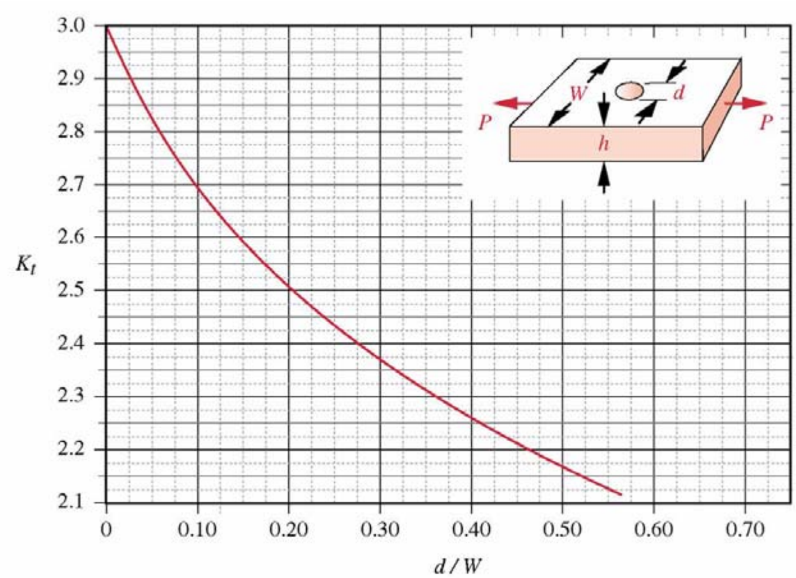


1) Material & Geometrical properties

From Table C-9 (p.1004) :

2) Applied Stresses

Stress Concentration :



for $\frac{d}{W} \leq 0.65$:

$$K_t \cong 3.0039 - 3.753\frac{d}{W} + 7.9735\left(\frac{d}{W}\right)^2 - 9.2659\left(\frac{d}{W}\right)^3 + 1.8145\left(\frac{d}{W}\right)^4 + 2.9684\left(\frac{d}{W}\right)^5$$

Figure E-13
Geometric Stress-Concentration Factor K_t for a Flat Bar with Transverse Hole in Axial Tension.

3) Fatigue Strength/Endurance Limit :

Load Factor : Eq. 6.7a -

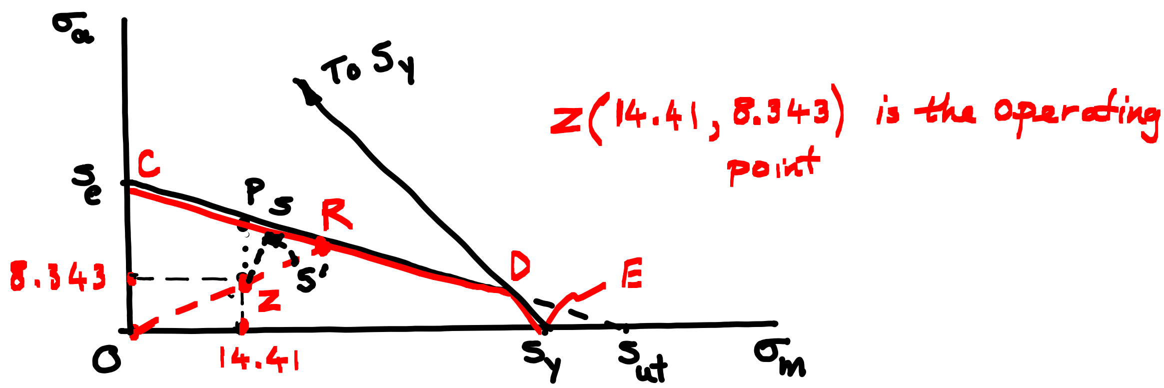
Size Factor : See sidenote on page 326 - For axial loading of any size cross-section :

Surface Factor :

Temperature :

Reliability :

$$S_e = C_{load}\, C_{size}\, C_{surf}\, C_{temp}\, C_{reliab}\, S_{e'}$$



The four possible safety factors can be calculated from Figs. 6-46 (p. 366). The smallest can be selected by examining the above figure (drawn more or less to scale), the plate does not fail under Case 1.

Case 2 :

$$N_f = \begin{cases} \dfrac{S_f}{\sigma_a'}\left(1 - \dfrac{\sigma_m'}{S_{ut}}\right) & \text{if } P \text{ is on } CD \\[2ex] \dfrac{S_y}{\sigma_a'}\left(1 - \dfrac{\sigma_m'}{S_y}\right) & \text{if } P \text{ is on } DE \end{cases}$$

Case 3 :

$$= \frac{S_f\, S_{ut}}{\sigma_a'\, S_{ut} + \sigma_m'\, S_f} \quad \text{if } R \text{ is on } CD \qquad (6.18e)$$

$N_f =$

[illegible]: $[illegible] = \dfrac{\overline{OS'}}{\overline{OZ}} = \dfrac{\overline{OZ} + \overline{ZS}}{\overline{OZ}}$ (6.18g)

where $\overline{OZ} = \sqrt{(\sigma_a')^2 + (\sigma_m')^2}$

$$\overline{ZS} = \sqrt{(\sigma_m' - \sigma_m'@S)^2 + (\sigma_a' - \sigma_a'@S)^2} \qquad (6.18f)$$

and where

$$\sigma_m'@S = \frac{S_{ut}(S_f^2 - S_f\sigma_a' + S_{ut}\sigma_m')}{S_f^2 + S_{ut}^2}$$

$$\sigma_a'@S = -\frac{S_f}{S_{ut}}(\sigma_m'@S) + S_f$$

As can be expected from the failure envelope diagram, the factor of safety corresponding to Case 4 is the smallest and should be selected as the factor of safety for fatigue failure of this plate. Since N_Y = 2.5, no localized yielding occurs and the threat is from fatigue.